

1)

$$\begin{aligned}
 f(x) &= x^3 + ax^2 + b & f'(x) &= 3x^2 + 2ax \\
 f(2) &= 8 + 4a + b & f'(2) &= 12 + 4a \\
 8 + 4a + b &= 3 & 12 + 4a &= 0 \\
 8 - 12 + b &= 3 & 4a &= -12 \\
 \boxed{b} &= 7 & \boxed{a} &= -3
 \end{aligned}$$

2)

$$\begin{aligned}
 f'(x) &= x(x-3)^2(x+1)^4 = 0 \\
 x &= 0 \quad x=3 \quad x=-1 \\
 f' & \begin{array}{c} - \quad - \quad + \quad + \\ \leftarrow \quad \quad \quad \rightarrow \\ -1 \quad 0 \quad 3 \end{array} \\
 f(x) & \text{ has ONE MIN @ } x=0
 \end{aligned}$$

3)

$y' < 0$  @ points C & D  
 b/c  $y$  is decreasing  
 $y'' < 0$  @ points A, B, C  
 b/c  $y$  is concave down  
 $\therefore y' < 0$  &  $y'' < 0$  @ point C

4)

$$\begin{aligned}
 y &= x^2 - 4x + 3 \quad [0, 5] \\
 \text{END point} & \quad y' = 2x - 4 \\
 \begin{array}{l|l} x & y \\ \hline 0 & 3 \\ 2 & -1 \\ 5 & 8 \end{array} \\
 x=0 & \quad 2x-4=0 \\
 x=5 & \quad x=2
 \end{aligned}$$

$y$  has abs max of  $y=8$  @  $x=5$ .

5)

$$\begin{aligned}
 y &= 2x^3 - 3x^2 - 12x \\
 y' &= 6x^2 - 6x - 12 = 0 \\
 x^2 - x - 2 &= 0 \\
 (x-2)(x+1) &= 0 \\
 x=2 \quad x=-1 \\
 y' & \begin{array}{c} + \quad - \quad + \\ \leftarrow \quad \quad \rightarrow \\ -1 \quad 2 \end{array} \\
 \bullet y & \text{ has a local max @ } x=-1 \\
 & \text{ b/c } y' \text{ \Delta s signs from + to -} \\
 \bullet y & \text{ has a local min @ } x=2 \\
 & \text{ b/c } y' \text{ \Delta s signs from - to +}
 \end{aligned}$$

6)

$$\begin{aligned}
 y &= x^4 - 4x^3 \\
 y' &= 4x^3 - 12x^2 \\
 y'' &= 12x^2 - 24x = 0 \quad y''' \\
 12x(x-2) &= 0 \\
 x=0 \quad x=2 & \quad \begin{array}{c} + \quad - \quad + \\ \leftarrow \quad \quad \rightarrow \\ 0 \quad 2 \end{array} \\
 y(0) &= 0 \quad y(2) = -16 \\
 y & \text{ has P.o.I @ } (0,0) \text{ & } (2,-16) \\
 & \text{ b/c } y'' \text{ \Delta s signs.}
 \end{aligned}$$

7)

$$\begin{aligned}
 f(x) &= x^4 - 4x^2 \\
 f'(x) &= 4x^3 - 8x = 0 \quad \begin{array}{c} + \quad - \quad + \\ \leftarrow \quad \quad \rightarrow \\ -\sqrt{2} \quad 0 \quad \sqrt{2} \end{array} \\
 4x(x^2 - 2) &= 0 \\
 x=0 \quad x=\pm\sqrt{2} \\
 (A) & \text{ 1 max, 2 min}
 \end{aligned}$$

8) point F, since  $f$  has a horizontal tangent and is concave down

9) point B, since  $f$  is increasing at a point of inflection

10) point G, since  $f$  is decreasing and concave down

11) point E, since  $f$  is increasing and concave up